Columbia University

Contributions to Education

Teachers College Series

STANDARDIZED REASONING TESTS IN ARITHMETIC AND HOW TO UTILIZE THEM

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PUBLISHED BY
Teachers College, Columbia University
NEW YORK CITY
1916

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CHAPTER I

INTRODUCTION

The occasion for publishing this booklet is the exhaustion of the edition of the book, "Arithmetical Abilities,"[1] in which the reasoning test herein contained first appeared. Then, too, it has been found that the brief record of the test as given in the original book does not give adequate help for those who may wish to utilize the test as a means of measuring the reasoning ability of children. It is hoped, then, that the publication of this booklet will furnish additional help to those who may wish to use this test both in supervision and in teaching.

Another purpose of publishing this book is that of furthering, in some measure, progress in the scientific study of the teaching of arithmetic. This progress will doubtless move along two lines, viz., the development and use of better and more complete tests, and the settling of some of the many problematic conflicts in the teaching of arithmetic.[2]

The test in "fundamentals" or formal operations as embodied in the book, "Arithmetical Abilities," has been so modified and improved by Courtis that it has been replaced by his *Series B*.[3] For the benefit of those who may wish to use it, the author's original test in fundamentals is here reproduced and permission is given for it to be reprinted so that each pupil may have a copy.

[1] Stone, Cliff W., Arithmetical Abilities and Some Factors Determining Them. This book is out of print but may be borrowed from libraries of teachers colleges or schools of education. Extended extracts may be found in Strayer's "Teaching Process"; Thorndike and Strayer's "Educational Administration"; Brown and Coffman's "How to Teach Arithmetic"; and Howell's "A Fundamental Study in the Teaching of Arithmetic."

[2] Some of these conflicts are stated in an article by the author entitled, Problems in Scientific Study of the Teaching of Arithmetic, *Journal of Educ. Psychology*, Vol. IV, No. 1, pp. 1-16.

[3] These tests may be secured from the author, S. A. Courtis, 81 Eliot St., Detroit, Michigan.

Test in Fundamentals

Work as many of these problems as you have time for; work them in order as numbered.

1. Add
   2375
   4052
   6354
   260
   5041
   1543

2. Multiply 3265 by 20.
3. Divide 3328 by 64.
4. Add
   596
   428
   94
   75
   302
   645
   984
   897

5. Multiply 768 by 604.
6. Divide 1918962 by 543.
7. Add
   4695
   872
   7948
   6786
   567
   858
   9447
   7499

8. Multiply 976 by 87.
9. Divide 2782542 by 679.
10. Multiply 5489 by 9876.
11. Divide 5099941 by 749.
12. Multiply 876 by 79.
13. Divide 62693256 by 859.
14. Multiply 96879 by 896.

The reasoning test is also reproduced here and those who wish to do so may have it reprinted so that each pupil may be provided with a copy. Those who prefer may secure copies, with a card of directions to pupils, in lots of fifty at the rate of 50 cents per package. Address: Bureau of Publications, Teachers College, Columbia University. Prices of quantities quoted upon application.

A second test, equivalent to the following, is being formulated and may be secured from the above address.

Reasoning Test

Solve as many of the following problems as you have time for; work them in order as numbered.

1. If you buy 2 tablets at 7 cents each and a book for 65 cents, how much change should you receive from a two-dollar bill?

2. John sold 4 Saturday Evening Posts at 5 cents each. He kept ½ the money and with the other ½ he bought Sunday papers at 2 cents each. How many did he buy?

3. If James had 4 times as much money as George, he would have $16. How much money has George?

4. How many pencils can you buy for 50 cents at the rate of 2 for 5 cents?

5. The uniforms for a baseball nine cost $2.50 each. The shoes cost $2 a pair. What was the total cost of uniforms and shoes for the nine?

6. In the schools of a certain city there are 2,200 pupils; ½ are in the primary grades, ¼ in the grammar grades, ⅛ in the high school and the rest in the night school. How many pupils are there in the night school?

7. If 3½ tons of coal cost $21, what will 5½ tons cost?

8. A newsdealer bought some magazines for $1. He sold them for $1.20, gaining 5 cents on each magazine. How many magazines were there?

9. A girl spent ⅛ of her money for carfare, and three times as much for clothes. Half of what she had left was 80 cents. How much money did she have at first?

10. Two girls receive $2.10 for making button-holes. One makes 42, the other 28. How shall they divide the money?

11. Mr. Brown paid ⅓ of the cost of a building; Mr. Johnson paid ½ the cost. Mr. Johnson received $500 more annual rent than Mr. Brown. How much did each receive?

12. A freight train left Albany for New York at 6 o'clock. An express left on the same track at 8 o'clock. It went at the rate of 40 miles an hour. At what time of day will it overtake the freight train if the freight train stops after it has gone 56 miles?

CHAPTER II

CONDITIONS AND DIRECTIONS FOR USING THE TEST

To get the greatest benefit from using a test the conditions under which it is given should duplicate those under which it was standardized. This test in reasoning is standardized in that it was given by the author to over three thousand pupils in one hundred and fifty-two classes of twenty-six representative school systems of the United States. These measurements were made in 1907 and 1908, and the test is further standardized in that it has been subsequently given in a number of representative school systems.

In the schools tested by the author, and presumably in the subsequent tests by others, the conditions were under such control that they were similar in each room of each school. A full statement of the precautions taken to maintain similar conditions is given in Part I of "Arithmetical Abilities."

The main points to be observed:

1. No announcement that a test is to be given should be made to pupils.
2. All directions to pupils should be given by the tester. (If the tester is a person from outside the system and if the teacher and principal are busy filling blanks with helpful data, the original conditions will be more fully duplicated.)
3. Principals and superintendents should refrain not only from communicating with pupils about the tests, but also from being present in the room either immediately before or during the test.
4. The time limit should be kept exactly—fifteen minutes to the second.
5. No mention should be made to the pupils of time limit.
6. Furnish each pupil with a copy of the test and have printed side turned down, until all begin.

7. Nothing should be said to pupils about the use of scratch paper, working in steps, amount of work to put down, etc.

How to give the test:

1. See to it that the conditions are a duplication of those stated above in so far as practicable.
2. Arrange for the test with the teacher of the pupils. (If you do not know her, it is best to have a note of introduction from the principal or superintendent.) Ask her to kindly help you by answering a set of questions headed, *Helpful Data furnished by the teacher at the time of the test.*[1]
3. Give either two or more of the Courtis tests in fundamental operations or the Stone test in fundamentals.[2]
4. Give the reasoning test. If Courtis tests precede, use *exactly the following* directions to pupils.[3] If the Stone test in fundamentals precedes, proceed as directed in footnote (2).

1. Take the materials that you usually take for your arithmetic work. Prepare two sheets of paper—headings and all. Have two sheets ready in case you may need them. *Use pencils.* Keep slip with printing turned down until we are ready to begin.
2. Now, do you have everything ready? In order for you to do your best in this test, you will need to do just as all the other boys and girls who have taken this test have done. So pay close attention and do just as I ask you to do.
3. You will not need to mark these (slips) papers at all. You will find directions at the top of these (slips) papers, telling you just what to do, so you will not need to ask any questions and—I do not think I need to say this to you, but I will, just because I have to all the other boys and girls,—be especially careful *not* to see anybody else's work. It is not easy not to see, but if you pay close attention to your own work only, the test will be the best.
4. Begin. (Allow exactly fifteen minutes.)

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[1] These questions may cover such items as length of time devoted to arithmetic, methods employed, etc. For further suggestions write C. W. Stone, Iowa State Teachers College, Cedar Falls, Iowa.

[2] If the Courtis tests are used, the directions contained in the folders accompanying them should be carefully followed. If the Stone test in fundamentals is given, follow directions given above under (4) except allow only 12 minutes. And after collecting the papers on fundamentals say:

1. Have two sheets prepared again. You may not need both but have them ready.
2. Keep the printing turned down until we are ready to begin—the same as before. Now are all provided?
3. Begin. (Allow exactly fifteen minutes.)

[3] In order that these directions may be followed verbatim it is best to have them copied on a card, that the tester may hold them before him while giving the test. These cards may be had from the Bureau of Publications, Teachers College, New York City, at 5 cents each.

CHAPTER III

DIRECTIONS FOR SCORING

In order to have the scores of a system, school, or pupil comparable with those in the scale as standardized, the papers should be marked and scored as were those in the original study. To this end it is essential that each of the following steps be taken according to directions:

(1) Mark the solutions on the basis of right or wrong reasoning. (2) Score the solutions according to assigned weighting and find scores for individual pupils. (3) Compute the total of the individual scores on the basis of a hundred pupils or compute the per cents of pupils making scores of 0, 1, 2, etc. (4) Compute the per cents of accuracy.

Directions for First Step

In marking the solutions the following rulings should be observed:

1. Mistakes in copying are not counted against the child; e.g.,

$$2 \times 7 = 14$$
$$14 \text{ and } 65 = 89$$
$$100 - 89 = 11$$

The reasoning of the problem was correct, so the mistake in copying is not counted.

2. Errors in fundamentals do not count against the reasoning score; e.g.,

\$2.50	\$5.50
\$2.00	9
\$5.50 cost of each.	\$50.50

The reasoning in this problem is correct, so the mistakes in fundamentals are not counted.

3. The child is given credit for the part of the problem that is reasoned correctly when part is wrong—the amount of credit depending upon the number of *steps* reasoned correctly; e.g.,

```
  7          65
  2         —14
 —           —
 14          41
```

Since one of the steps is correct, this was counted 1/3 correct.

4. If a problem is unfinished, credit is given for the steps taken if they are correct, or in so far as they are correct; e.g.,

(a) 2200 pupils.
1/2 of 2200 is 1100 pupils in primary grades.

This was counted 1/5 correct.

```
(b)  $2.50 cost of one uniform.
         9
    ------
    $22.50 cost of nine uniforms.
     $2.00 cost of one pair shoes.
         9
    ------
    $18.00 cost of nine pair shoes.
```

This was counted 2/3 correct.

Directions for Second Step

The weightings to be used in scoring the solutions are:

For	first	problem,	1
"	second	"	1
"	third	"	1
"	fourth	"	1
"	fifth	"	1
"	sixth	"	1.4
"	seventh	"	1.2
"	eighth	"	1.6
"	ninth	"	2
"	tenth	"	2
"	eleventh	"	2
"	twelfth	"	2

For example, any one of the first five problems solved correctly should score 1 for the pupil, school, and system; the sixth would count 1.4; the seventh, 1.2, etc. In the third illustration under Directions for First Step, the solution of the first problems as given would count ⅓ for the pupil, school, and system; and in the first illustration under the 4th ruling the partial solution of the sixth problem would count 1/5 of 1.4 or .28.

The score of each individual pupil is found by adding all the scores of all the solutions found on his paper.

Directions for Third Step

Sort the papers, placing the smallest score on top, and graduate so that the largest is on the bottom.

To get a single measure of a class, school, or system, add scores of all pupils and compute on basis of a hundred pupils; e. g., Mr. Hebden found the score for 716 Baltimore city pupils to be 5749.48. Reduced to the basis of a hundred pupils this is 803.2 as the score for the Baltimore system. (This is the measure used in the Springfield, Illinois, Survey and by Supt. Stark in the Hackensack Report. See graphs in Chapter IV.)

To get the most helpful measure for *supervision,* compute the per cents of pupils that made scores of 0, 1, 2, 3, 4, etc., and construct a surface of distribution showing deviation from the standard. (This is the plan followed in the Butte, Montana, Survey and in handling the Iowa State Teachers College Training School scores. See graphs in Chapter IV.)

The most helpful measure for *teaching* is the score of each individual pupil. (Illustrations are shown at the end of Chapter IV in the representation of the scores of the VI-A-I class of the Bloomington, Indiana, schools and in the VI-B class of the Iowa State Teachers College Training School.)

Directions for Fourth Step

The per cent of accuracy is found by computing the per cent reasoned correctly. In going over the papers for the data for this figuring, the following rulings should be observed:

1. In determining the number of problems attempted, count number on which any work was done. This will be sufficiently exact for fifty to a hundred pupils. But for determining the number of problems attempted by individual pupils, if the last problem worked is incomplete, use fractions to represent the steps attempted.
2. In determining the number of problems correct for fifty to a hundred pupils, count the number that are worked entirely correct. But for computing the scores of individual pupils, use fractions to represent the steps correct.

CHAPTER IV

REPRESENTING, INTERPRETING, AND UTILIZING SCORES

The degree of benefit derived from giving the test will turn on the effectiveness with which the results are utilized. One of the essentials for full utilization is effective representation. There are many effective means of representing scores and the one which a given person uses should be the one that is most effective for that person. For some, this will be tables of scores; for others, surfaces of distribution; for others, graphs of progress. Simple graphs are not difficult to construct and a little practice will help most persons to use them to great advantage. Whatever scheme is used the result should be that the scores are so placed that they may be readily and fully interpreted.

Whether tabular or graph representation is used the standings of the twenty-six systems tested for the study, "Arithmetical Abilities," will be needed. Given by Roman numerals instead of by names these systems, with their scores, are:

System	XXIII,	score	356
"	XXIV	"	429
"	XVII	"	444
"	IV	"	464
"	XXV	"	464
"	XXII	"	468
"	XVI	"	469
"	XX	"	491
"	XVIII	"	509
"	XV	"	532
"	III	"	533
"	VIII	"	538
"	VI	"	550
	Median	"	*551*
"	I	"	552
"	X	"	601
"	II	"	615
"	XXI	"	627
"	XIII	"	636

System	XIV	score	661
"	IX	"	691
"	VII	"	734
"	XII	"	736
"	XI	"	759
"	XXVI	"	791
"	XIX	"	848
"	V	"	914

The above tabulation is on the basis of a hundred pupils to each system and the first step in representing scores is to reduce them to that basis, e.g., seventy-five pupils in the Bloomington schools made a combined score of 539.65 in 1914. Reduced to the basis of a hundred pupils this is a score of 720. See graph of Bloomington scores, page 15, as an illustration of the use of this table.

Accuracy in work is quite as important as *amount* of work, hence, besides the scale of scores showing the gross achievements, there is need of the scale of accuracy. The following is the table of per cents of problems correct in the twenty-six systems originally tested. See graph of Bloomington records, page 20.

System	XVI	54.9%	correct
"	XVII	55.1%	"
"	XXIII	58.9%	"
"	III	63.3%	"
"	I	65.3%	"
"	XV	66.3%	"
"	VI	68.2%	"
"	XXII	69.1%	"
"	X	70.3%	"
"	XXIV	71.1%	"
"	XIII	71.2%	"
"	XXVI	71.4%	"
"	VIII	72.1%	"
"	XVIII	73.0%	"
"	XIX	73.6%	"
"	XXV	74.7%	"
"	IV	74.9%	"
"	XX	76.6%	"
"	VII	76.9%	"
"	XIV	77.1%	"
"	XII	77.7%	"
"	IX	79.7%	"
"	II	80.3%	"
"	V	81.4%	"
"	XXI	84.3%	"
"	XI	85.6%	"

A simple though not especially effective representation is that of placing the score of a given system with that of other representative systems. The following is an illustration of this representation as used in the Springfield, Illinois, Survey:

TABLE 23.—SCORE PER EACH 100 PUPILS IN REASONING IN ARITHMETIC IN SPRINGFIELD AND 26 OTHER SCHOOL SYSTEMS

	Lowest	Middle	Highest	Springfield	Springfield's rank from top
Reasoning	356	550	914	508	19
Accuracy	55	72	86	70	19

The figures in the last column of the table show that in both the amount of work accomplished and the accuracy with which it was done the Springfield children rank in the 19th place among the 27 systems compared. That is to say, they are more than two-thirds of the way down the list.

The *purpose* of the representation is the best criterion for deciding which of the various schemes for representing to employ. The main purposes for which scores are represented are to aid supervision and to aid teaching. Superintendents and other supervisors responsible for the work of many teachers need to be able to see readily the status of a class as a whole; teachers, on the other hand, need to be able to see readily the status of pupils as individuals.

REPRESENTING SCORES AS AIDS IN SUPERVISION

For purposes of supervision the ideal representation is that which conveys the class status with the minimum of time and effort. One of the best plans is that of the per cent distribution according to scores. This is illustrated in the portrayal of the Butte, Montana, reasoning scores (Fig. I). Fig. I represents the percentage of children making the given scores in reasoning problems. For example, 19 per cent of the fifth grade children made score of 0; 19 per cent made score of 1; etc. The lines representing the median scores for each grade tell about how many in each grade surpass the median scores for the grades above, and how many fall below the median scores for the grades below.

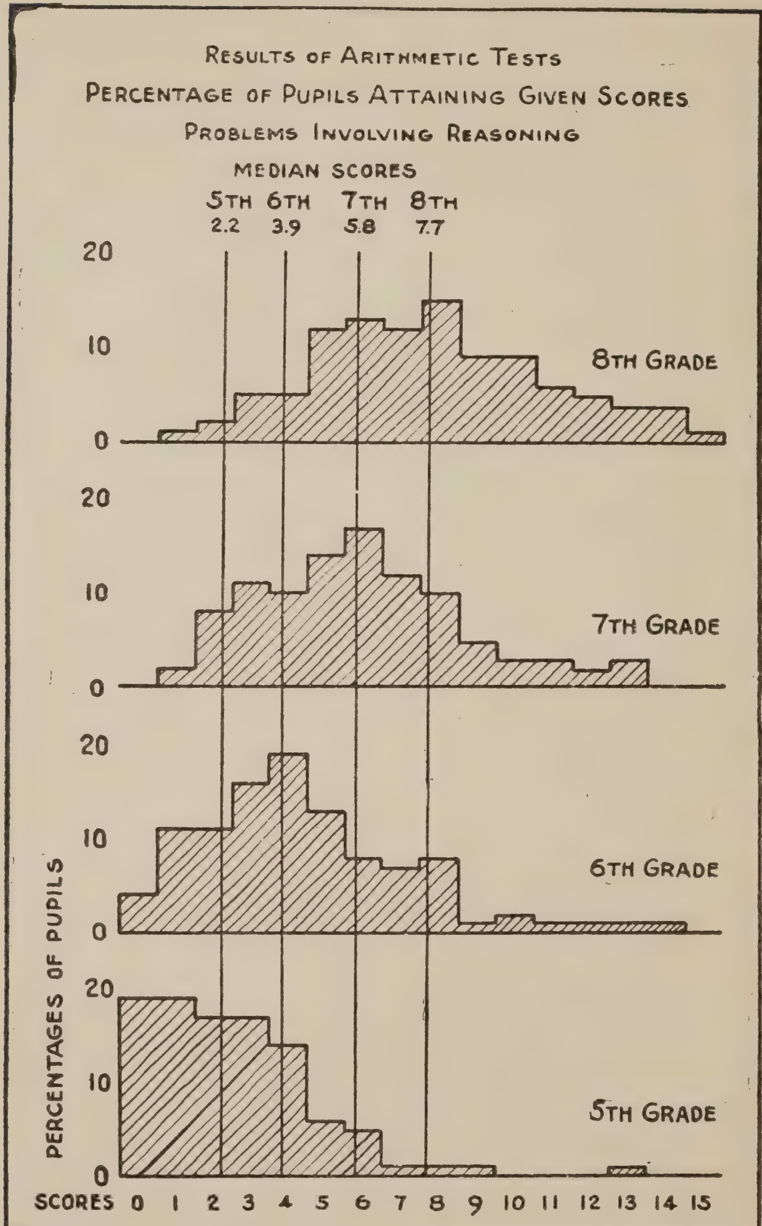

FIG. I. Representing percentage of children making the given scores in reasoning problems. Butte, Montana.

Representation by graphs will be facilitated by using coördinate paper of suitable rulings. The following form shows a good size. Copies of this form on coördinated paper may be secured from the Bureau of Publication, Teachers College, Columbia University.

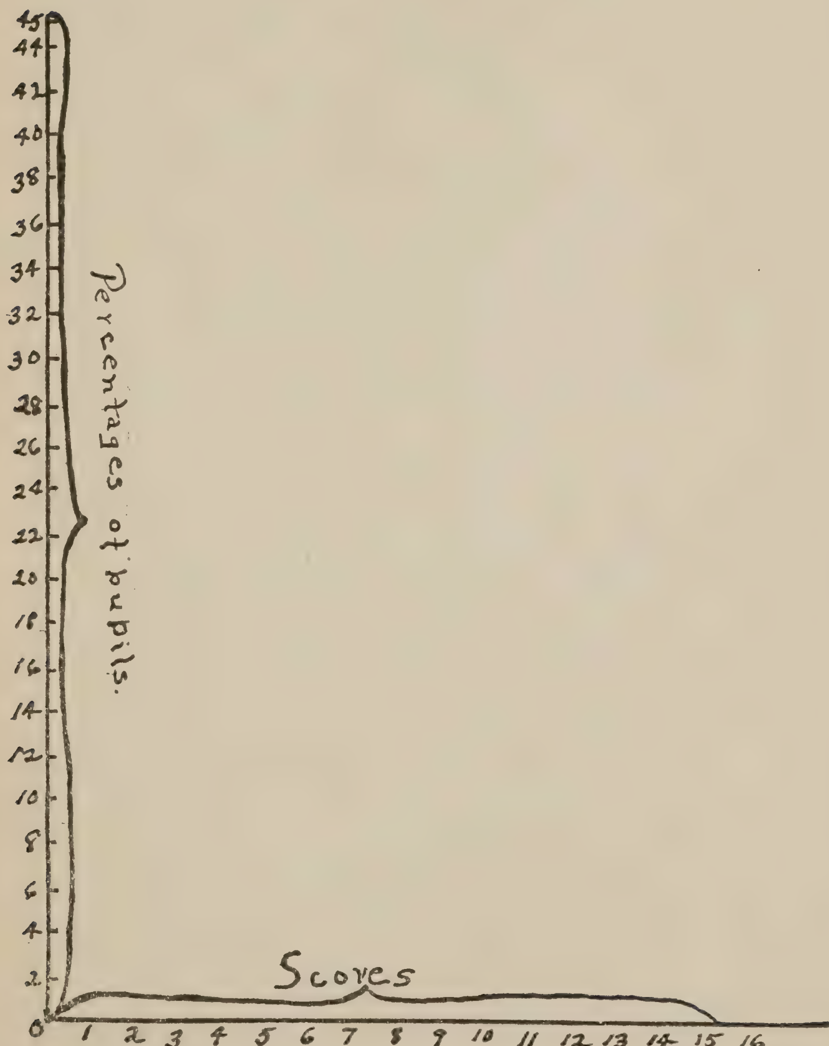

FIG. II. Graph form for representing percentages of pupils attaining given scores.

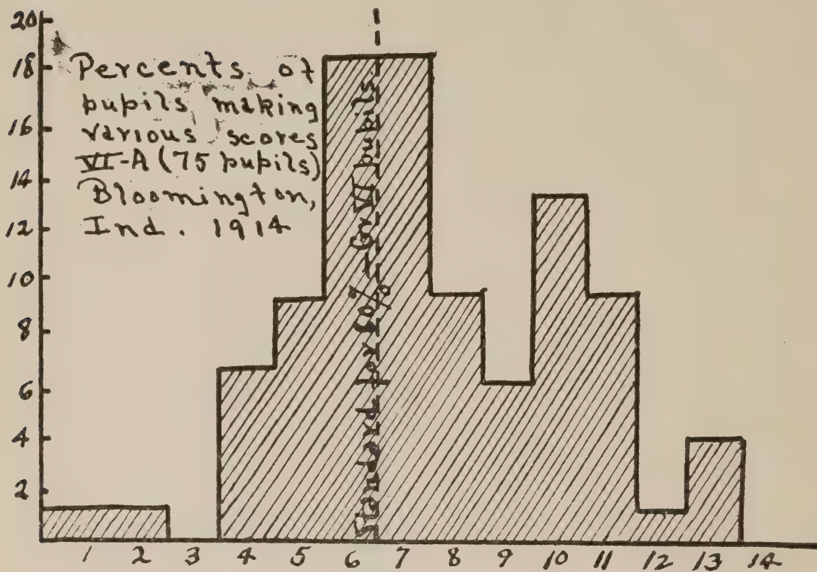

FIG. III. Percentages of VI-A (75 Bloomington, Indiana) pupils making various scores.

The procedure in filling out a form to represent the percentages of a given system or class may be explained in connection with Figs. II and III. Following the directions as previously given (page 8) the papers were sorted and the following table derived for the 1914 VI A pupils of Bloomington, Indiana:

Number	grading	0— .5,	1= 1.3%	of the 75 pupils.
"	"	.5— 1.5,	1= 1.3%	" " " "
"	"	1.5— 2.5,	1= 1.3%	" " " "
"	"	2.5— 3.5,	0= 0.0%	" " " "
"	"	3.5— 4.5,	5= 6.6%	" " " "
"	"	4.5— 5.5,	7= 9.3%	" " " "
"	"	5.5— 6.5,	14=18.6%	" " " "
"	"	6.5— 7.5,	14=18.6%	" " " "
"	"	7.5— 8.5,	7= 9.3%	" " " "
"	"	8.5— 9.5,	4= 5.3%	" " " "
"	"	9.5—10.5,	10=13.3%	" " " "
"	"	10.5—11.5,	7= 9.3%	" " " "
"	"	11.5—12.5,	1= 1.3%	" " " "
"	"	12.5—13.5,	3= 4.0%	" " " "

By comparing this table with the graph it will be seen that one pupil or 1.3% of the 75 pupils whose score was 0-.5 is

represented in the lower left-hand corner of the form; the 1.3% with a score of .5—1.5, is represented next; the 1.3% with a score of 1.5—2.5 next; and as there were no pupils receiving a score of 2.5—3.5 there is no representation on the form; but 6.6% had a score of 4 (3.5–4.5) and the graph extends above 4 to 6.6; the 9.3% with a score of 5 are represented above 5, etc. The vertical broken line at 6.5 indicates the Standard. Thus the supervisor can see at a glance (1) the percentage of pupils attaining each score, and (2) the percentages that are up to or above Standard.

Another way of representing the data for supervisory purposes is illustrated in the following graph of scores of the Bloomington, Indiana, schools where the author has tested the reasoning abilities of pupils for a series of years.

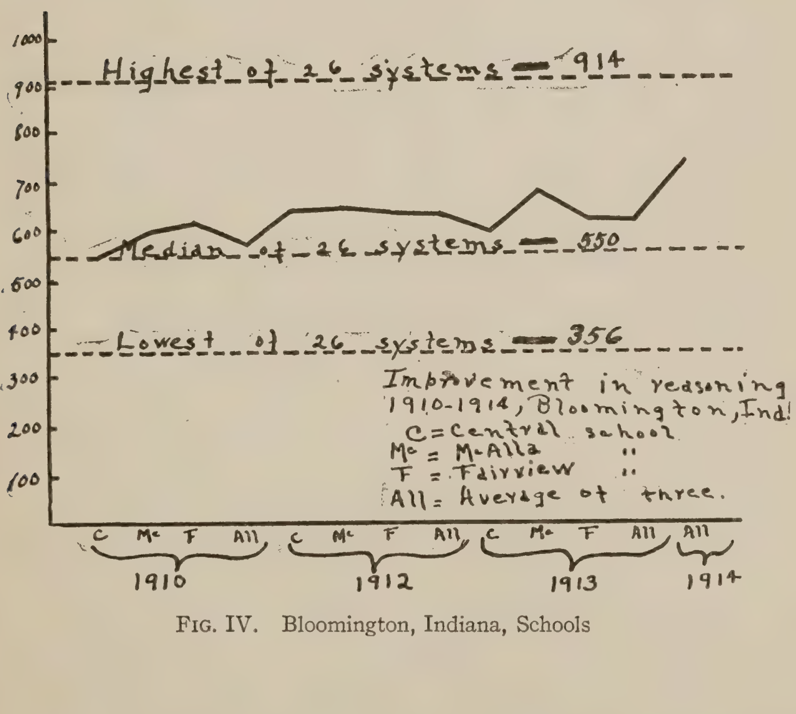

FIG. IV. Bloomington, Indiana, Schools

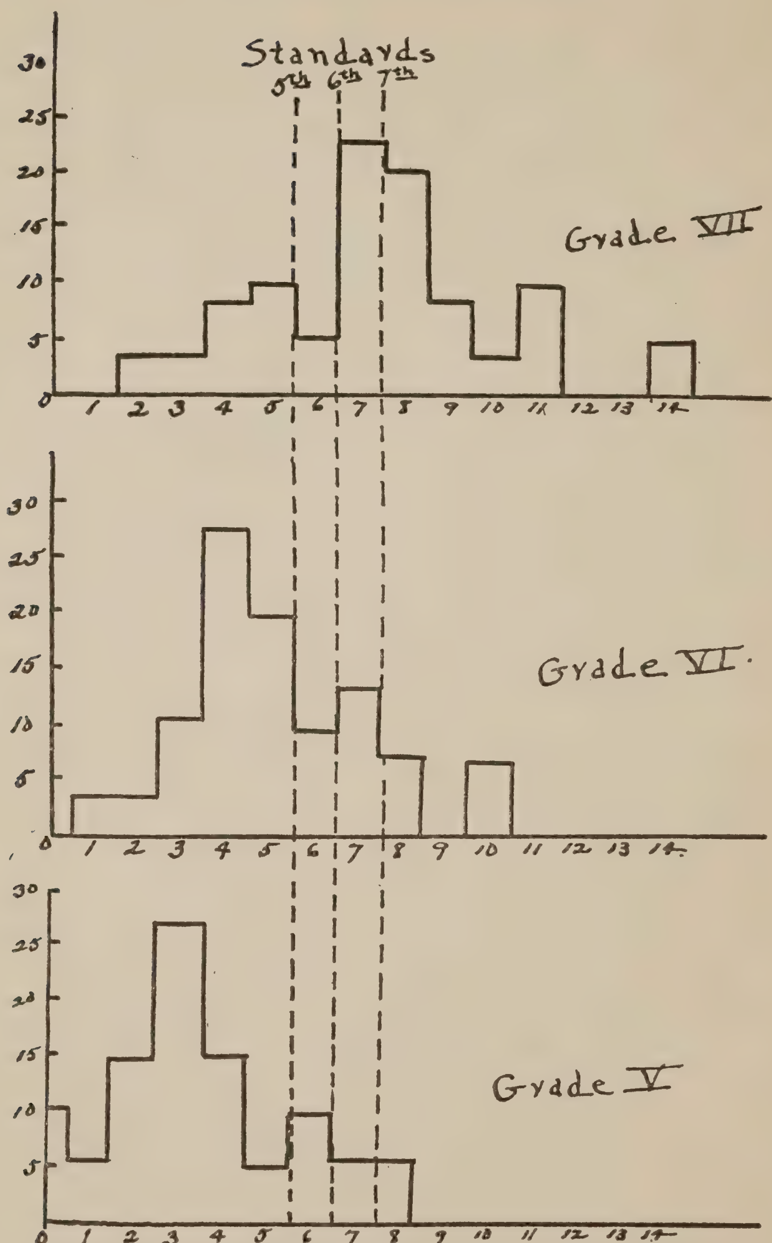

FIG. V. Reasoning abilities Indiana State Teachers College Training School, January, 1916. This representation enabled the supervisor to see at a glance where the pupils of each of these classes stood in reasoning at the midyear and to know what percentage of each class needed special attention during the second semester.

Another markedly effective illustration is that employed by Supt. Stark of the Hackensack, New Jersey, schools:

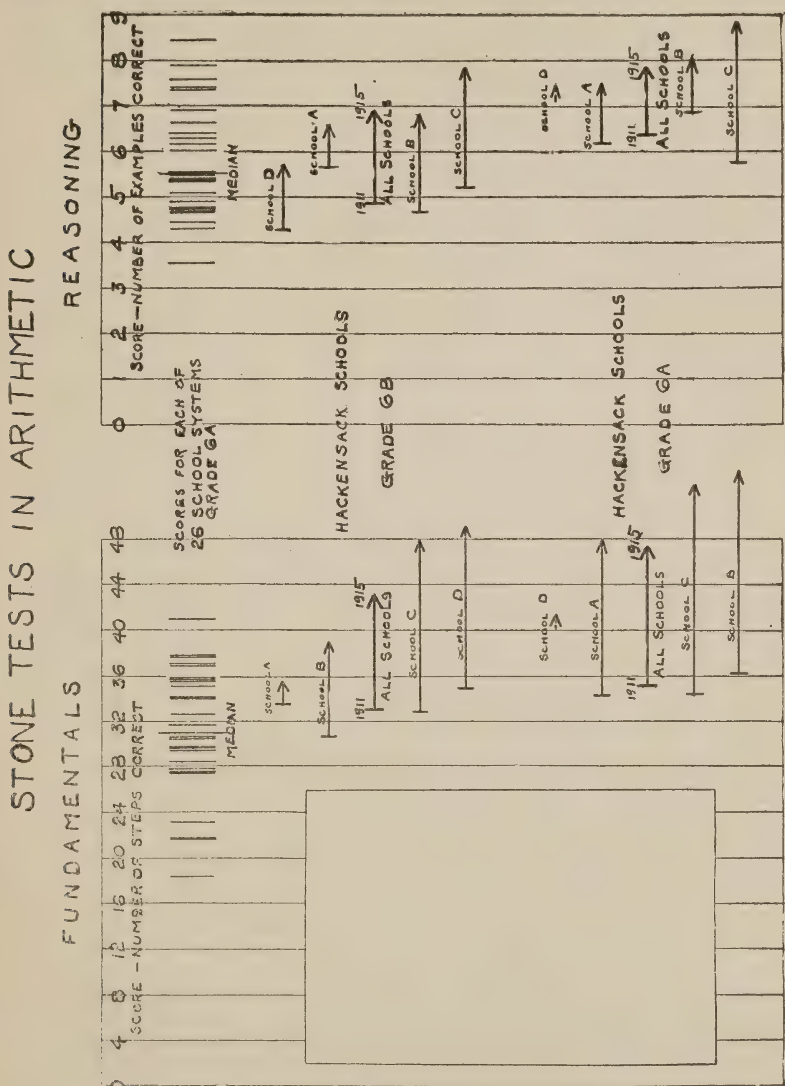

Fig. VI

Representing Scores as Aids in Teaching

For purposes of teaching, the ideal representation is that which conveys the status of individual pupils with the minimum of

time and effort. As far as the author has found, the best form of graphs for this purpose is that which shows the exact status of each individual pupil. As the supervisor needs to see by classes, so the teacher needs to see by individuals. The following are illustrations:

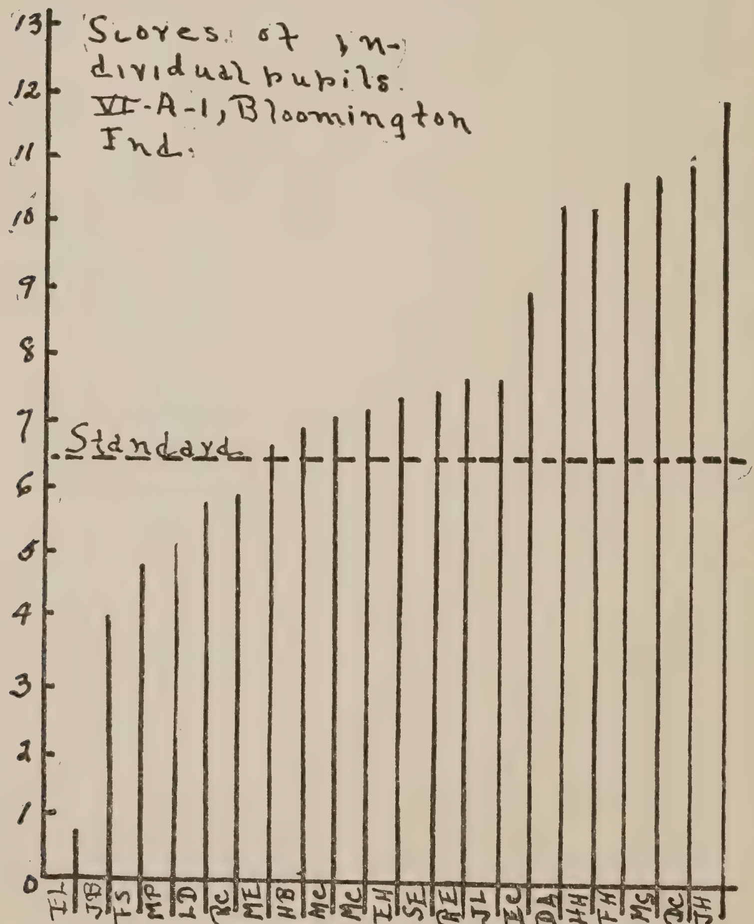

FIG. VII. Each line stands for the score of a pupil. For purposes of identification the initials of the pupils are placed at the bottom, e.g., J.H. not only reached Standard, but did so well that his line extends almost twice as high as the Standard of the VI A Grade requires. This form of graph brings out vividly just where each pupil stands, e.g., E.L. with a score of less than 1.

Coördinated paper enables one to make such graphs readily and accurately.

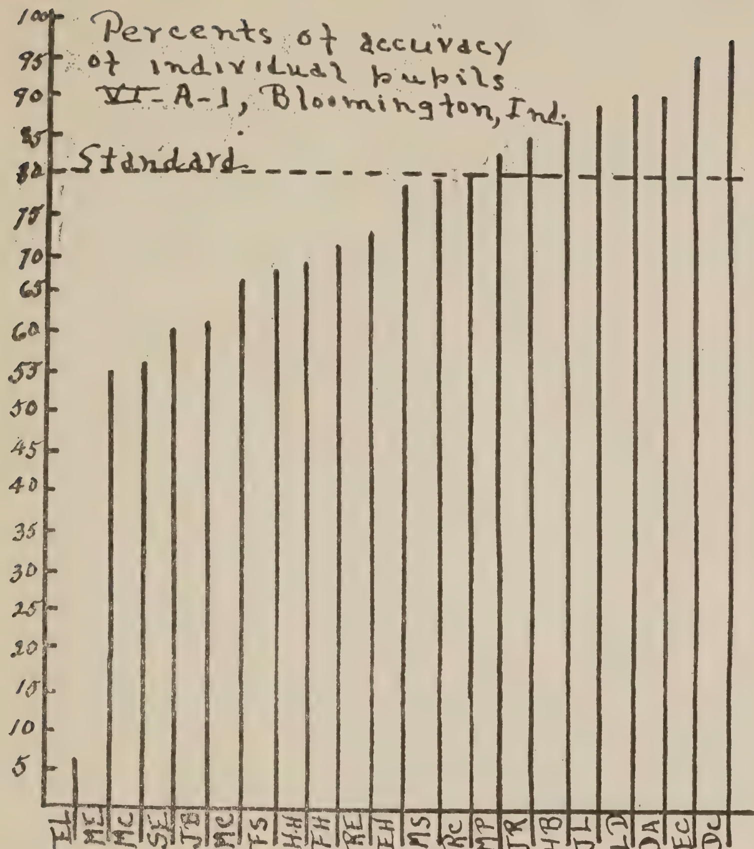

FIG. VIII. Here the lines stand for per cents of accuracy for individual pupils, e.g., D.C. is well above the Standard in accuracy, while E.L. is far below.

INTERPRETING SCORES

The scores having been represented effectively, the next important step in the use of the test is now possible, viz., that of interpreting the results. This is preëminently the responsibility and privilege of the teacher. To do it successfully requires the most intimate knowledge of the individual pupils.

The interpretation of results implies an agreement as to standards, and unfortunately this is one of the unsolved problems of education. Probably one of the most satisfactory bases

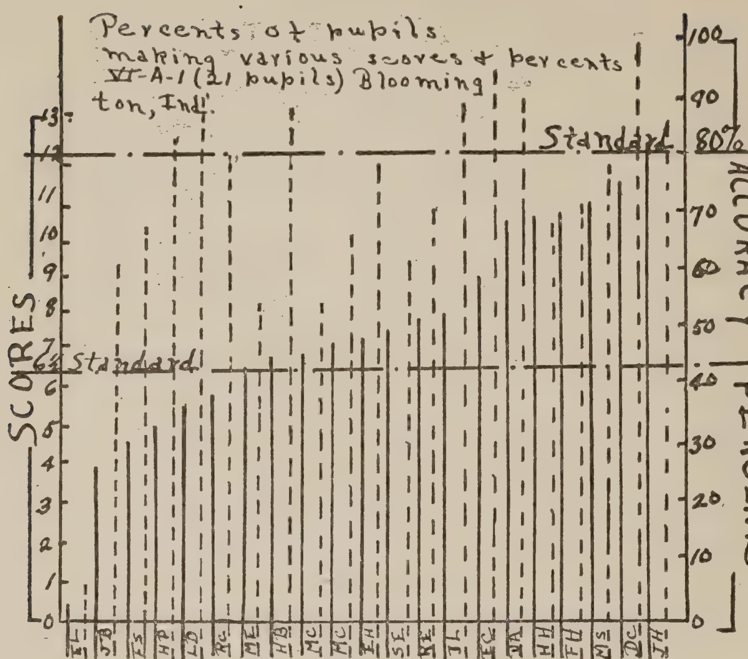

FIG. IX. This is a representation of both scores and accuracy for individual pupils. It enables the teacher to see at a glance the status of each pupil in both amount and quality of work, e.g., J.H. is seen to be above Standard in both, while L.D. is markedly accurate but does not reach the Standard in score. This shows the teacher that L.D.'s difficulty is lack of speed and she is in a position to determine the cause of the slowness and to set about removing it.

now possible for proposing standards is the achievements of the better schools of the United States. Whether teachers should consider their pupils up to standard if 50 per cent of them reach or surpass the median ability shown by the twenty-six representative systems or whether they should strive to have 80 per cent of them attain that ability, is a question which the school workers of each system must settle for themselves. The important point is that such an agreement as to standards is now possible and that it is very beneficial. The author offers the following as tentative standards for grades V to VIII.

Tentative Standards

That 80 per cent or more of 5th grade pupils reach or exceed a score of 5.5 with at least 75 per cent accuracy; that 80 per cent or more of 6th grade pupils reach or exceed a score of 6.5 with at least 80 per cent accuracy; that 80 per cent or more of 7th grade pupils reach or exceed a score of 7.5 with at least 85 per cent accuracy; that 80 per cent or more of 8th grade pupils reach or exceed a score of 8.75 with at least 90 per cent accuracy.

Having decided where pupils ought to be, the results of the tests will show how many of them are there and then it remains for the teacher and her supervisor to decide (1) how many others can and ought to progress towards that goal, (2) how far each can probably go, (3) how best to help them in that progress, and (4) how best to help those who have *arrived* to make the best use of their time.

Two Dangers

In interpreting the results of a test, care should be taken to avoid two dangers: (1) *that of placing too much reliance on a single measure of an individual.* (No pupil whose score is surprisingly low should be so graded without a second trial. The first test may have come on an "off day" for the individual. See Fig. X); (2) *that of relying on the average of class scores to show whether the pupils are "up to standard."* (It may be that there are a few very high scores and a few very low scores, and both these undesirable extremes will be hidden in the average. It is always best to study the scores of individual pupils and if a single measure of ability is needed, to use the *median.*)

Diagnosing Causes of Low Scores

The causes of failure or success are many and often complex. In the results of the mental processes of reasoning it is especially difficult to be certain of the exact causes. In diagnosing cases

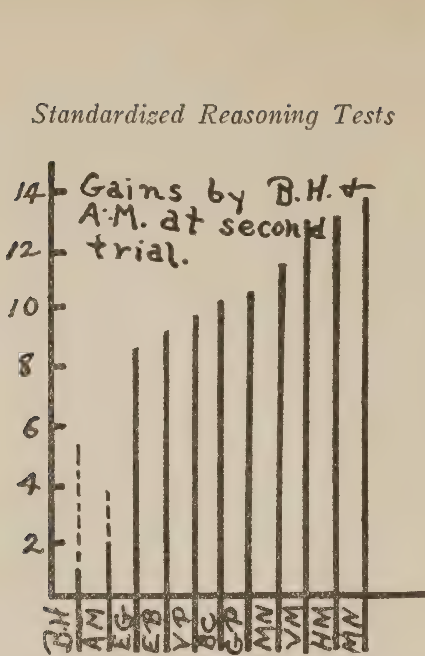

FIG. X. The score of B. H. was unexpectedly low and to have diagnosed her case on the basis of the first test would have been inaccurate and unjust.

of failure the following are some possible causes that may well be considered:

1. Inability to read.—Any pupil who is poor in reasoning should be thoroughly tested by the use of some of the measures of silent reading.
2. Poor judgment as to the amount of writing to do in the solution.—Much time and energy are lost by labored labels and elaborate indications of steps.
3. Physical disability.—Poor eyesight is a common hindrance in this as well as other phases of school work.
4. Lack of physical coördination.—Tests in copying figures and other coördination tests should be used.
5. Inability to see relationships between steps.—This is frequently an innate lack and very difficult to overcome.
6. Mental laziness.—There is no spur equal to knowing just where one stands and where one ought to expect to stand.
7. Lack of a realization of the passing of time.
8. Temporary disability.—Individuals have "off days" and surprisingly low scores should not be regarded as the measure of an individual without a second trial.[1]

[1] But as the first scores are counted in making the scale, it is best that *all* pupils be included in the scores as computed for the class or system.

Prescribing for Improvement

A record of the treatment of certain individuals[1] with low scores will serve to illustrate the possibility of prescribing for improvement. The records will be given under *diagnosis, treatment* and *results.*

Pupil, H. C.

Diagnosis: Up to standard in reading ability, did not indulge in undue labeling, physical examination showed no defects, constantly made low scores. Conclusion as to cause of low score: *Mental laziness with lack of realization of the passing of time.*

Treatment: The pupil was first of all made conscious of his status by comparing his score with those of his fellow classmates and with the standard; then he was helped to study his way of working which convinced him of the seat of his difficulty. From day to day lists of approximately equivalent problems were given him with time limit. Much was made of record of scores, gain being expected by both teacher and pupil.

Results: Within a few days notable gain appeared, due to increased ability to direct and hold attention to the work in hand. Contrasted with his previous tendency to wander, the pupil became capable of working continuously in spite of such distractions as people entering the room. After about twenty minutes daily for three weeks he raised his score from 4 to 5.4. Though this is not a large gain in score, the boy had made it largely of his own initiative; he had formed an ideal of concentration, and the concept of *giving attention* to reasoning processes was well under way. It is believed by those who have studied the boy that much of his improvement was due to the convincingness of the objective evidence of his need to improve.

Some Pupils of a Certain Fifth Grade

Diagnosis: Many pupils made very low scores, many papers much covered with such statements as, " If one tablet cost 7 cents, 2 tablets . . . etc." Here was evidently one large source of failure.

Treatment: Emphasis was placed on the possibility of saving time by not writing so much, brief labels were devised, originality was encouraged, and approval of pupils and teacher placed on briefest adequate statement.

Results: As shown by second test and by daily work, much time was saved for reasoning processes. The following parallel columns show typical results.

Pupil, A. K.

In first test	*In second test*
They would cost $18.	$2.50 × 9 = $22.50
If one suit cost $2.50, 9 would cost $2.50 × 9 = $22.50.	$2 × 9 = $18.
They would cost $40.50.	$40.50.
Score in first test, **1 1-3.**	Score in second test, **3,**

[1] This record of pupils was furnished by Miss Floe E. Correll, supervising critic of mathematics, Iowa State Teachers College Training School.

Pupil, L. I. C.

In first test	*In second test*
If he sold 4 papers and got twenty cents for them, one-half would be 10 cents and with the other 10 cents he bought Sunday papers, he would buy as many as 2 will go into 10 or 2|10 5 papers.	5 4 — 20 One-half would be 10 cents and he could buy five.
Score in first test, **3,**	Score in second test, **4 1-2,**

Equivalent Tests

Equivalent tests are desirable for measuring the results of treatment or lapses of time. Reasoning tests equivalent to the one included in this book, are being constructed by the author. They will be published and made available by Teachers College Bureau of Publications. Special care is being taken to have these tests strictly equivalent to the original; and the procedure in utilizing them will be the same as set forth in this book for the original test.